

# CHARACTERIZING RIGID SIMPLICIAL ACTIONS ON TREES

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**ABSTRACT.** We extend Forester's rigidity theorem so as to give a complete characterization of rigid group actions on trees (an action is rigid if it is the only reduced action in its deformation space, in particular it is invariant under automorphisms preserving the set of elliptic subgroups).

Let  $T$  be a simplicial tree with a cocompact action of a group  $G$  (i.e. the Bass-Serre tree associated to a decomposition of  $G$  as a finite graph of groups). A  $G$ -tree  $T'$  is a *deformation* of  $T$  if it may be obtained from  $T$  by a finite sequence of expansions and collapses (elementary moves coming from the canonical isomorphism  $A *_B B \simeq A$ ). These moves do not change the set of elliptic subgroups (a subgroup is *elliptic* if it fixes a point in the tree). Conversely, Forester proved [3] that any cocompact  $G$ -tree  $T'$  with the same elliptic subgroups as  $T$  is a deformation of  $T$ .

Since an expansion makes the tree more complicated, and a collapse makes it simpler, it is natural to restrict to reduced trees. A tree  $T$  is *reduced* if one cannot perform a collapse on  $T$ . Equivalently,  $T$  is reduced if, whenever an edge  $e$  has the same stabilizer as one of its endpoints, then both endpoints of  $e$  are in the same  $G$ -orbit (i.e.  $e$  projects onto a loop in the quotient graph).

The tree  $T$  is *rigid* if it is reduced, and it is the only reduced tree in its deformation space (up to equivariant isomorphism). In other words, all deformations of  $T$  (trees with the same elliptic subgroups as  $T$ ) may be reduced to  $T$  by collapse moves. Rigidity provides a canonical element  $T_{red}$  in the deformation space; in particular, any automorphism of  $G$  that preserves the set of elliptic subgroups leaves  $T_{red}$  invariant (see [1, 4, 5, 7, 8, 9] for examples and applications to JSJ splittings and automorphisms).

Forester proved that “strongly-slide-free” trees are rigid ([3], see also [6]). The purpose of this note is to extend Forester's theorem. Our extension is optimal: we obtain a complete characterization of rigid trees.

Before stating our result, let us illustrate it on generalized Baumslag-Solitar groups [2, 9]. Note that these groups have been classified up to quasi-isometry [10]. The rigidity we are studying here is not quasi-isometric rigidity of groups, as the group is fixed once and for all.

We consider a finite graph of groups with each vertex and edge group isomorphic to  $\mathbf{Z}$ . It is pictured as a labelled graph, each label being the index of the edge

group in the vertex group (see figure 1). [One should allow negative labels, but we will not bother here.]

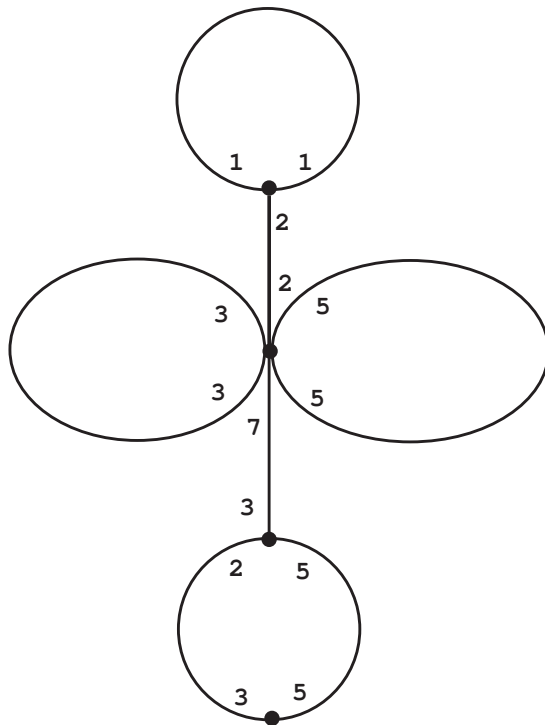


FIGURE 1. A graph of groups whose universal cover is a rigid tree

The associated Bass-Serre tree  $T$  is reduced if and only if the label 1 appears only on loops (edges with both endpoints equal). It is strongly slide-free (in the sense of [3]) if and only if there is no divisibility relation: a divisibility relation is a relation  $p \mid q$ , where  $p, q$  are labels at the same vertex.

We will show that the tree associated to a labelled graph is rigid if and only if the graph is as on figure 1. Namely, a divisibility relation  $p \mid q$  at a vertex  $x$  is allowed if  $p = q$  and  $p, q$  are carried by one loop. If  $x$  has valence 3, with a loop labelled  $(1, 1)$  and a third label  $r$ , we allow the relations  $1 \mid r$ . No other divisibility relation is allowed.

There is one exception to the statement just given. The tree associated to the standard presentation of the solvable Baumslag-Solitar group  $BS(1, s)$  as an HNN-extension is rigid if and only if  $s = 1$  or  $s$  is prime.

We shall now give the general statement. Let  $T$  be a  $G$ -tree. If  $e$  is an oriented edge,  $\bar{e}$  denotes  $e$  with the opposite orientation. We always assume that  $G$  acts without inversions. We denote by  $G_v$  (resp.  $G_e$ ) the stabilizer of a vertex  $v$  (resp. an edge  $e$ ).

A  $G$ -tree  $T$  is (associated to) an *ascending HNN-extension* if the quotient graph of groups has one vertex and one edge, and furthermore at least one of the two monomorphisms from the edge group to the vertex group is onto. If  $T$  is reduced,

this is equivalent to the existence of a  $G$ -fixed end in  $T$ , and also to the length function being the absolute value of a homomorphism  $G \rightarrow \mathbf{R}$ .

**Theorem 1 (general case).** *Let  $T$  be a reduced cocompact  $G$ -tree which is not an ascending HNN-extension. It is rigid if and only if, given two oriented edges  $e, f$  with the same origin  $v$  such that  $G_e \subset G_f$ , one of the following holds:*

- (1)  $e$  and  $f$  are in the same  $G_v$  orbit.
- (2)  $e$  and  $\bar{f}$  are in the same  $G$ -orbit, and  $G_e = G_f$ .
- (3) there is an edge  $f'$  with origin  $v$ , in the same  $G$ -orbit as  $\bar{f}$ , such that  $G_f = G_{f'} = G_v$ . Furthermore, there are only three  $G_v$ -orbits of edges with origin  $v$  (those of  $e, f, f'$ ).

Strongly slide-free trees are those for which only (1) occurs.

**Theorem 2 (exceptional case).** *Let  $T$  be the Bass-Serre tree of an ascending HNN-extension  $G = \langle A, t \mid tat^{-1} = \varphi(a) \rangle$ , with  $\varphi : A \rightarrow A$  an injective homomorphism.*

*The tree  $T$  is rigid if and only if, for every subgroup  $H \subset A$  containing  $\varphi(A)$ , there exist  $i, j \geq 0$  and  $a_0, a_1 \in A$  such that  $a_0$  conjugates  $\varphi^i(A)$  to  $\varphi^j(H)$ , and  $\varphi(a_0^{-1})a_0\varphi^{i+1}(a_1)$  centralizes  $\varphi^{i+1}(A)$ .*

If  $\varphi(A)$  is either  $A$  or a maximal proper subgroup of  $A$ , the tree is rigid. The converse holds when  $A$  is abelian.

Theorem 1 is proved in the first two sections. Theorem 2 is proved in the third one.

## RIGIDITY

We prove the “if” direction of Theorem 1. Let  $T'$  be reduced, with the same elliptic subgroups as  $T$ . We show  $T' = T$ .

• First assume that case (3) in the statement of Theorem 1 does not occur. Our arguments generalize those of [6].

We define a map  $f : T \rightarrow T'$  in the following way. We choose a representative  $v_i$  for each orbit of vertices of  $T$ , and we let  $f(v_i)$  be a vertex of  $T'$  fixed by  $G_{v_i}$ . We then extend  $f$  to a  $G$ -equivariant map sending vertex to vertex, and linear on edges.

Since  $T$  is as in Theorem 1, and case (3) does not occur, no vertex stabilizer  $G_v$  of  $T$  can fix an edge. As in [6, p. 324], it follows that distinct vertices of  $T$  have distinct images in  $T'$ . Also note that the stabilizer of  $f(v)$  equals the stabilizer of  $v$ .

The key point is to show that there is no folding, a fold being a pair of edges  $e_1, e_2$  with origin  $v$  such that  $f(e_1) \cap f(e_2)$  is a non-degenerate segment.

Suppose there is a fold. Let  $w'$  be the vertex of  $T'$  adjacent to  $f(v)$  on  $f(e_1) \cap f(e_2)$ . The argument in the proof of Lemma 2.1 in [6] shows that the stabilizer  $G_{w'}$  of  $w'$  cannot be contained in  $G_v$ . Since  $G_{w'}$  is elliptic, it fixes a vertex  $w \neq v$  in  $T$ . Let  $e$  be the initial edge of  $vw$ .

Any element of  $G_v$  fixing  $w'$  also fixes  $e$ . In particular,  $G_e$  contains both  $G_{e_1}$  and  $G_{e_2}$ . Since (1) or (2) holds for each of the pairs  $(e_1, e)$  and  $(e_2, e)$ , we know that  $e_1, e_2, e$  are in the same orbit as non-oriented edges. In particular,  $f(e_1)$  and  $f(e_2)$  have the same length.

Say that the fold between  $f(e_1)$  and  $f(e_2)$  is of type (1) if  $e_1, e_2$  are in the same  $G_v$ -orbit. In this case  $G_{e_1}$  and  $G_{e_2}$  are properly contained in  $G_e$ , because elements of  $G_v$  taking  $e_1$  to  $e_2$  belong to  $G_e$  (they fix  $w'$ ). In particular,  $e$  is in the same  $G_v$ -orbit as  $e_1$  and  $e_2$ .

If the fold is of type (2) (i.e.  $e_1$  and  $\bar{e}_2$  are in the same  $G$ -orbit), we claim that  $G_{e_1} = G_{e_2}$ . We may assume that  $e_1$  and  $e$  are in the same  $G_v$ -orbit. Since  $G_{e_2} \subset G_e$ , we have  $G_{e_2} = G_e$ . But then  $G_{e_1} \subset G_{e_2}$ , and finally  $G_{e_1} = G_{e_2}$ . Also note that  $f(e_1)$  and  $f(e_2)$  are folded along strictly less than half their (common) length: otherwise, an element of  $G$  sending  $e_1$  to  $\bar{e}_2$  would fix a point in  $T'$  but not in  $T$ .

We can now extend Lemma 2.2 of [6]:

**Lemma.** *If  $e_1, e_2, e_3$  are three consecutive (non-oriented) edges in  $T$ , then  $f(e_1) \cap f(e_2) \cap f(e_3) = \emptyset$ .*

*Proof.* Denote  $v_1 = e_1 \cap e_2$  and  $v_2 = e_2 \cap e_3$ . There is a problem only if there are folds both at  $v_1$  and at  $v_2$ . If both folds are of type (1), the argument of [6] applies. If both folds are of type (2), we simply use the fact that less than half is folded. We complete the proof by ruling out the possibility of mixed folds: type (1) at  $v_1$ , type (2) at  $v_2$ .

Orient  $e_1, e_2, e_3$  so that  $e_1$  and  $e_2$  have origin  $v_1$ , and  $e_3$  has origin  $v_2$ . They are in the same  $G$ -orbit as oriented edges. Let  $e_0$  be the image of  $e_2$  by a group element taking  $e_3$  to  $e_1$ . Its terminal endpoint is  $v_1$ . Since the fold at  $v_2$  is of type (2), we have  $G_{e_2} = G_{e_3}$ , and therefore  $G_{e_0} = G_{e_1}$ .

Now consider an edge  $e$  (with origin  $v_1$ ) associated to the fold  $e_1, e_2$  as above. It is in the  $G_{v_1}$ -orbit of  $e_1$  (but not of  $\bar{e}_0$ ). Furthermore  $G_e$  properly contains  $G_{e_1}$ , hence also  $G_{e_0}$ . This shows that  $\bar{e}_0, e$  do not satisfy any of conditions (1) or (2) of the theorem, a contradiction.  $\square$

Deducing  $T' = T$  from this lemma is as in [6, pp. 326-327].

- We now have to allow case (3) in the statement of Theorem 1 to occur. Let  $e, f, f', v$  be as in case (3). Since  $T$  is reduced,  $G_e$  is properly contained in  $G_v$ . It follows that the normalizer of  $G_v$  is a semi-direct product  $N(G_v) = G_v \rtimes \langle t \rangle$ . It acts on a line  $L_v \subset T$  (containing  $f$  and  $f'$ ), with  $G_v$  acting as the identity and  $t$  as a unit translation (taking  $\bar{f}$  to  $f'$ ). Every point of  $L_v$  has stabilizer  $G_v$ .

We claim that the translation axis  $L'_v$  of  $t$  in  $T'$  has similar properties. First,  $G_v$  is the identity on  $L'_v$  because  $G_v$  is elliptic in  $T'$  and  $t$  normalizes  $G_v$ . Since  $G_v$  is a maximal elliptic subgroup, points of  $L'_v$  have stabilizer equal to  $G_v$ . In particular,  $gL'_v \cap L'_v = \emptyset$  if  $g \notin N(G_v)$ . Adjacent vertices on  $L'_v$  are in the same  $G$ -orbit because  $T'$  is reduced. They are in the same orbit under  $t$  because  $N(G_v)$  is generated by  $G_v$  and  $t$ , so  $t$  acts on  $L'_v$  as a unit translation.

For each vertex  $v$  as in case (3) of Theorem 1 (there may be several  $G$ -orbits of them), collapse  $L_v$  to a point. Similarly, collapse each  $L'_v$  (noting that collapsed lines are pairwise disjoint). We obtain reduced  $G$ -trees  $T_0, T'_0$  with the same elliptic subgroups: those of  $T$ , and subgroups of an  $N(G_v)$ . Furthermore,  $T_0$  satisfies the hypothesis of Theorem 1, with only cases (1) and (2) occurring. Thus  $T_0 = T'_0$ .

To reconstruct  $T$  and  $T'$  from  $T_0$  and  $T'_0$ , one has to blow up certain points into lines. Since these points project onto terminal vertices in the quotient graph  $T_0/G$ , there is only one way of blowing up. This shows  $T' = T$ .

### DEFORMATIONS

Given a  $G$ -tree  $T$ , a *collapse move* consists in choosing an edge  $e = vu$  such that  $v, u$  are in distinct orbits and  $G_e = G_u$ , and collapsing each edge in the orbit of  $e$  to a point. The stabilizer of  $v$  has not changed since  $G_v *_{G_u} G_u = G_v$ . In the quotient graph of groups, one has collapsed an edge to a point. An *expansion move* is the opposite operation.

Now we consider *slide moves* (see [3] or [4]). Suppose that  $T$  contains adjacent edges  $e, f$  with origin  $v$  such that  $G_e \subset G_f$ , and  $e, f$  are not in the same  $G$ -orbit as non-oriented edges. We can then slide  $e$  across  $f$ , so that it now starts at the terminal endpoint of  $f$ . Doing this  $G$ -equivariantly replaces  $T$  by another  $G$ -tree  $T_1$ .

**Lemma.** *Let  $T$  be reduced. If  $e, f$  are not as in case (3) of Theorem 1, then sliding  $e$  across  $f$  does change  $T$  (the new tree  $T_1$  is not equivariantly isomorphic to  $T$ ).*

*Proof.* We may assume that  $T_1$  is reduced. We show that the translation length of some element of  $G$  changes.

Let  $u$  (resp.  $w$ ) be the terminal endpoint of  $e$  (resp.  $f$ ). If there is  $g \in G$  sending  $v$  to  $u$ , the result is clear because the translation length of  $g$  goes from 1 to 2. Suppose there is no such  $g$ . In particular,  $G_e \subsetneq G_u$  since  $T$  is reduced. Fix  $g_u \in G_u \setminus G_e$ .

If  $G_f \subsetneq G_v$ , choose  $h \in G_v \setminus G_f$ . Note that  $h \notin G_e$  (because  $G_e \subset G_f$ ). The translation length of  $g_u h$  is twice the distance between the fixed point sets of  $g_u$  and  $h$ , so passes from 2 to 4. The case  $G_f \subsetneq G_w$  is similar, so we assume  $G_f = G_v = G_w$ .

If we are not in case (3) of Theorem 1, there is an edge  $e_0 = vv_0$  not in the same  $G$ -orbit as  $e$  or  $f$  (as a non-oriented edge). If there exists  $g_0 \in G_{v_0} \setminus G_{e_0}$ , the translation length of  $g_0 g_u$  goes from 4 to 6. If  $G_{e_0} = G_{v_0}$ , choose  $h$  with  $h v_0 = v$ . The translation length of  $g_u h$  goes from 3 to 5 (the translation length of  $h$  is 1, and the distance from its axis to the fixed point set of  $g_u$  goes from 1 to 2).  $\square$

We now prove the “only if” direction of Theorem 1, by deforming  $T$  into a reduced tree  $T'$  different from  $T$ . Consider adjacent edges  $e, f$ , with  $G_e \subset G_f$ , satisfying none of the three conditions of Theorem 1. There are two cases.

If  $e$  and  $\bar{f}$  are not in the same orbit, we can change  $T$  within its deformation space by sliding  $e$  across  $f$ . The new tree is reduced, except if  $G_e = G_f = G_w \subsetneq$

$G_v$ , where  $w$  is the terminal endpoint of  $f$ . If this happens, we choose  $t \in G$  taking  $w$  to  $v$  and we slide  $e$  across  $t\bar{f}$  rather than across  $f$ .

The second possibility is that  $e$  and  $\bar{f}$  are in the same orbit and  $G_e \subsetneq G_f$ . We may assume  $G_f \subsetneq G_v$ : otherwise  $T$  is an ascending HNN-extension (if every edge with origin  $v$  is in the  $G_v$ -orbit of  $e$  or  $f$ ), or there exist edges as in the previous case (if there is a third orbit).

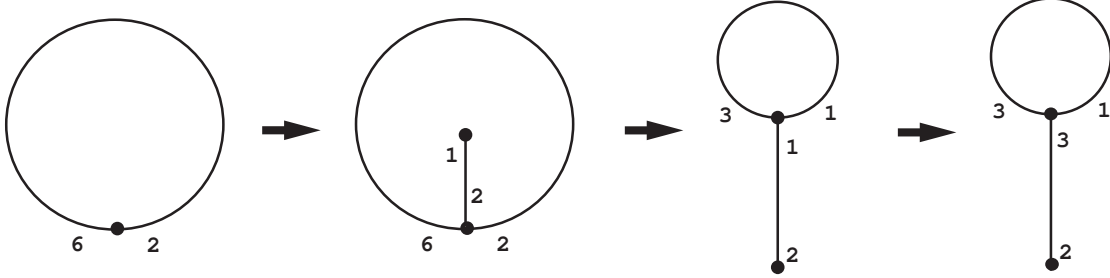


FIGURE 2. Expanding and sliding in  $BS(2, 6)$

We first perform an expansion move at  $v$ , creating a new edge with origin  $v$  and stabilizer  $G_f$ . In the quotient graph of groups, we have created an edge and a terminal vertex, both with group  $G_f$ . We then slide  $e$  and  $f$  across the new edge (the case of  $BS(2, 6)$  is illustrated on figure 2). In the graph of groups, the terminal vertex now has valence 3, and exactly two of the three edge groups map onto  $G_f$  (the tree is not reduced). The last step is to slide the new edge across  $f$  (counterclockwise on figure 2). This yields a reduced tree because  $G_e \subsetneq G_f \subsetneq G_v$ .

#### ASCENDING HNN-EXTENSIONS

We shall say that a  $G$ -tree  $T_1$  is a *reduction* of a  $G$ -tree  $T_2$  if it is reduced and may be obtained from  $T_2$  by performing collapse moves. We also say that the quotient graph of groups  $T_1/G$  is a reduction of  $T_2/G$ . Every cocompact  $G$ -tree (resp. every finite graph of groups) has at least one reduction.

Consider a graph of groups  $\Gamma$  of the following form. It is a subdivided circle, consisting of two vertices  $v, v'$  and two edges  $e, e'$ , and both inclusions  $G_e \hookrightarrow G_v$  and  $G_{e'} \hookrightarrow G_{v'}$  are onto. Such a graph of groups has two reductions, obtained by collapsing either edge. Both are ascending HNN-extensions, and we say that the associated Bass-Serre trees are related by an *induction move* (through  $\Gamma$ ).

**Lemma.** *Let  $T$  be an ascending HNN-extension. If  $T'$  is a reduced tree in the deformation space of  $T$ , then  $T'$  is an ascending HNN-extension and it may be obtained from  $T$  by a finite number of induction moves.*

*Proof.* Join  $T'$  to  $T$  by a sequence  $S_n$ , with  $S_{n+1}$  obtained from  $S_n$  by an expansion or a collapse move. For each  $n$ , choose a reduction  $T_n$  of  $S_n$ . Thus  $T_n$  and  $T_{n+1}$  are two different reductions of the same tree ( $S_n$  or  $S_{n+1}$ ). We complete the proof by

showing that two reductions of a tree  $S$  in the deformation space of  $T$  are related by an induction move (and are ascending HNN-extensions).

Since  $S$  may be obtained from  $T$  by expansions and collapses, one can check that the graph of groups  $\Gamma_S$  associated to  $S$  has the following form. It consists of a (subdivided) circle  $C$ , possibly with trees  $C_i$  attached to vertices  $v_i \in C$ . The circle  $C$  may be oriented so that, if  $e = vw$  is an oriented edge of  $C$ , then  $G_e = G_v$ . Furthermore, the fundamental group of  $C$  (viewed as a subgraph of groups) equals  $G$  (i.e. the fundamental group of  $C_i$  equals  $G_{v_i}$ ).

It follows from this description that any reduction of  $S$  is an ascending HNN-extension, obtained by choosing an edge  $e \subset C$  and collapsing all other edges. Now consider two reductions of  $S$ . They are associated to edges  $e, e' \subset C$ , and they differ by an induction move (through the graph of groups  $\Gamma$  obtained by collapsing all edges of  $\Gamma_S$  except  $e$  and  $e'$ ).  $\square$

**Corollary.**  *$T$  is rigid if and only if  $T' = T$  for every  $T'$  related to  $T$  by an induction move.*  $\square$

Let  $T$  be associated to the ascending HNN-extension  $G = \langle A, t \mid tat^{-1} = \varphi(a) \rangle$ , with  $\varphi : A \rightarrow A$  an injective homomorphism. It contains an edge  $e = vw$ , with  $G_v = A$ ,  $G_e = G_w = \varphi(A)$ , and  $w = tv$ .

Let  $T'$  be related to  $T$  by an induction move through  $\Gamma$ . Let  $T_0$  be the Bass-Serre tree of  $\Gamma$ . In  $T_0$ , the segment between  $v$  and  $w$  consists of two edges  $vv_0$  and  $v_0w$ . The stabilizer of  $v_0w$  is  $\varphi(A)$ , and the stabilizer of  $vv_0$  is a group  $H$  with  $\varphi(A) \subset H \subset A$ . The tree  $T'$  (obtained from  $T_0$  by collapsing edges in the orbit of  $v_0w$ ) is the Bass-Serre tree  $T_H$  associated to the presentation of  $G$  as the “induced” HNN-extension  $G = \langle H, t \mid tht^{-1} = \varphi|_H(h) \rangle$ . Conversely, if  $\varphi(A) \subset H \subset A$ , the tree  $T_H$  is related to  $T$  by an induction move.

This shows that  $T'$  is related to  $T$  by an induction move if and only if  $T'$  is a  $T_H$ , with  $\varphi(A) \subset H \subset A$ . Proving Theorem 2 now reduces to showing that  $T_H = T$  if and only if there exist  $i, j, a_0, a_1$  as in the statement of the theorem.

The tree  $T_H$  is characterized (up to  $G$ -equivariant isomorphism) by the existence of an edge  $e' = v'w'$  with  $G_{v'} = H$  and  $G_{e'} = G_{w'}$ , and an element  $t' \in G$  sending  $v'$  to  $w'$  such that  $t'ht'^{-1} = \varphi(h)$  for  $h \in H$ .

If  $T_H = T$ , view  $v'w'$  as an edge of  $T$  and fix  $g \in G$  taking the “base edge”  $vw$  to  $v'w'$ . Recall that  $G_v = A$  and  $w = tv$ . The elements taking  $v'$  to  $w'$  are those of the form  $gta_1g^{-1}$ , with  $a_1 \in A$ . Thus  $T_H = T$  if and only if there exist  $g \in G$  and  $a_1 \in A$  such that

$$\begin{cases} gAg^{-1} = H \\ (gta_1g^{-1})gag^{-1}(gta_1g^{-1})^{-1} = \varphi(gag^{-1}) \quad \text{for } a \in A. \end{cases}$$

Any  $g \in G$  may be written  $g = t^{-j}a_0t^i$ , with  $i, j \geq 0$  and  $a_0 \in A$ . Since  $t^iAt^{-i} = \varphi^i(A)$  and  $t^jHt^{-j} = \varphi^j(H)$ , one has  $gAg^{-1} = H$  if and only if  $a_0\varphi^i(A)a_0^{-1} = \varphi^j(H)$ .

The other equation then becomes

$$t^{-j}a_0t^i ta_1aa_1^{-1}t^{-1}t^{-i}a_0^{-1}t^j = \varphi(t^{-j}a_0t^i at^{-i}a_0^{-1}t^j).$$

We rewrite it as

$$a_0\varphi^{i+1}(a_1aa_1^{-1})a_0^{-1} = t^j\varphi(t^{-j}a_0\varphi^i(a)a_0^{-1}t^j)t^{-j}.$$

Since  $a_0\varphi^i(a)a_0^{-1} \in \varphi^j(A)$ , the right-hand side equals  $\varphi(a_0\varphi^i(a)a_0^{-1})$ , and we get

$$a_0\varphi^{i+1}(a_1)\varphi^{i+1}(a)\varphi^{i+1}(a_1^{-1})a_0^{-1} = \varphi(a_0)\varphi^{i+1}(a)\varphi(a_0)^{-1}.$$

This expresses that  $\varphi(a_0^{-1})a_0\varphi^{i+1}(a_1)$  centralizes  $\varphi^{i+1}(A)$ , and concludes the proof of Theorem 2.

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